Propositional Logic

2.1 Symbols and Translation

2.2 Truth Functions

2.3 Truth Tables for Propositions

2.4 Truth Tables for Arguments

2.5 Indirect Truth Tables
## 2.1 Symbols and Translation Notes

<table>
<thead>
<tr>
<th>Operator</th>
<th>Name</th>
<th>Logical Function</th>
<th>Used to Translate</th>
</tr>
</thead>
<tbody>
<tr>
<td>~</td>
<td>tilde</td>
<td>negation</td>
<td>not, it is false that, it is not the case that</td>
</tr>
<tr>
<td>*</td>
<td>dot</td>
<td>conjunction</td>
<td>and, also, but, moreover, however, nevertheless, still, both, additionally, furthermore</td>
</tr>
<tr>
<td>V</td>
<td>wedge</td>
<td>disjunction</td>
<td>or, unless</td>
</tr>
<tr>
<td>⊃</td>
<td>horseshoe</td>
<td>implication</td>
<td>if...then..., only if, given that, provided that, in case, on condition, that, sufficient condition for, necessary condition for</td>
</tr>
<tr>
<td>=</td>
<td>triple bar</td>
<td>equivalence</td>
<td>if and only if, is a necessary and sufficient condition for</td>
</tr>
</tbody>
</table>

**Tips for translation:**

Remember that translation from ordinary English to logical expressions often results in a "distortion of meaning" because not all ordinary statements can be adequately expressed in logical terms.

Be sure you can locate the **main operator** in a statement. Parenthesis can point us to the main operator. If you are not sure which operator functions as the main operator, please reread this section.

There are five types of operators:

- **Negation**
- **conjunction** or conjunctive statements
- **disjunction** or disjunctive statements
- **conditionals** that express material implication (antecedent and consequent)
- **biconditionals** that express material equivalence

**Conditional statements do not always translate in an intuitive manner.** Sometimes it will be necessary to switch the order of the subject and predicate terms to get an accurate translation.

- **Necessary and sufficient conditions:** be sure you know the differences highlighted

  When translating a conditional proposition, place the sufficient condition in the antecedent position and the necessary condition in the consequent position.

- **Biconditionals** are used to translate the phrase "is a necessary and sufficient condition for" and should not be confused with conditional statements. In order to be translated as a biconditional, the proposition in question must serve as both a necessary and sufficient condition.

  When there are more than two simple propositions (i.e., two letters), it is necessary to group phrases together for an accurate translation. Look for clues given by commas, semicolons, and grouping phrases like "either," "both," etc.
2.1 Symbols and Translation Exercises Part I

Using the letters E, I, J, L, and S to abbreviate the simple statements, “Egypt’s food shortage worsens,” “Iran raises the price of oil,” “Jordan requests more U. S. aid,” “Libya raises the price of oil,” and “Saudi Arabia buys five hundred more warplanes,” symbolize these statements.

1. Iran raises the price of oil but Libya does not raise the price of oil.
2. Either Iran or Libya raises the price of oil.
3. Iran and Libya both raise the price of oil.
4. Iran and Libya do not both raise the price of oil.
5. Iran and Libya both do not raise the price of oil.
6. Iran or Libya raises the price of oil but they do not both do so.
7. Saudi Arabia buys five hundred more warplanes and either Iran raises the price of oil or Jordan requests more U. S. aid.
8. Either Saudi Arabia buys five hundred more warplanes and Iran raises the price of oil or Jordan requests more U. S. aid.
9. It is not the case that Egypt’s food shortage worsens, and Jordan requests more U. S. aid.
10. It is not the case that either Egypt’s food shortage worsens or Jordan requests more U. S. aid.
11. Either it is not the case that Egypt’s food shortage worsens or Jordan requests more U. S. aid.
12. It is not the case that both Egypt’s food shortage worsens and Jordan requests more U. S. aid.
14. Unless Egypt’s food shortage worsens, Libya raises the price of oil.
15. Iran won’t raise the price of oil unless Libya does so.
2.1 Symbols and Translation Exercises Part II

Symbolize the following, using capital letters to abbreviate the simple statements involved.

1. If Argentina mobilizes, then if Brazil protests to the UN, then Chile will call for a meeting of all the Latin American states.

2. If Argentina mobilizes, then either Brazil will protest to the UN or Chile will call for a meeting of all the Latin American states.

3. If Argentina mobilizes, then Brazil will protest to the UN and Chile will call for a meeting of all the Latin American states.

4. If Argentina mobilizes, then Brazil will protest to the UN, and Chile will call for a meeting of all the Latin American states.

5. If Argentina mobilizes and Brazil protests to the UN, then Chile will call for a meeting of all the Latin American states.

6. If either Argentina mobilizes or Brazil protests to the UN, then Chile will call for a meeting of all the Latin American states.

7. Either Argentina will mobilize or if Brazil protests to the UN, then Chile will call for a meeting of all the Latin American states.

8. If Argentina does not mobilize, then either Brazil will not protest to the UN or Chile will not call for a meeting of all the Latin American states.

9. If Argentina does not mobilize, then neither will Brazil protest to the UN nor will Chile call for a meeting of all the Latin American states.

10. It is not the case that if Argentina mobilizes, then both Brazil will protest to the UN, and Chile will call for a meeting of all the Latin American states.

11. If it is not the case that Argentina mobilizes, then Brazil will not protest to the UN, and Chile will call for a meeting of all the Latin American states.

12. Brazil will protest to the UN if Argentina mobilizes.

13. Brazil will protest to the UN only if Argentina mobilizes.

14. Chile will call for a meeting of all the Latin American states only if both Argentina mobilizes and Brazil protests to the UN.

15. Brazil will protest to the UN only if either Argentina mobilizes or Chile calls for a meeting of all the Latin American states.
2.1 Symbols and Translation Exercises Part III

Translate the following statements into symbolic form using capital letters to represent affirmative English statements.

1. Cartier does not make cheap watches.

2. Arizona has a national park but Nebraska does not.

3. Either Stanford or Tulane has an architecture school.

4. Both Harvard and Baylor have medical schools.

5. If Chanel has a rosewood fragrance, then so does Lanvin.

6. Chanel has a rosewood fragrance if Lanvin does.

7. Maureen Dowd writes incisive editorials if and only if Paul Krugman does.

8. Reese Witherspoon wins best actress only if Martin Scorsese wins best director.

9. Armani will launch a leather collection given that Gucci rejects skinny models.

10. The Colts’ winning most of their games implies that Peyton Manning is a great quarterback.

11. Bill Gates does not support malaria research unless Warren Buffet does.

12. Mercedes will introduce a hybrid model only if Lexus and BMW do.

13. Mariah Carey sings pop and either Elton John sings rock or Diana Krall sings jazz.

14. Either Mariah Carey sings pop and Elton John sings rock or Diana Krall sings jazz.

15. Not both Jaguar and Porsche make motorcycles.


17. Either Nokia or Seiko makes cell phones.

18. Not either Ferrari or Maserati makes economy cars.

19. Neither Ferrari nor Maserati makes economy cars.

20. Either Ferrari or Maserati does not make economy cars.

21. If Glenn Beck spins the news, then if Keith Olberman fights back, then Rachel Maddow tells it straight.

22. If Glenn Beck’s spinning the news implies that Keith Olberman fights back, then Rachel Maddow tells it straight.

23. Tommy Hilfiger celebrates casual if and only if neither Ralph Lauren nor Calvin Klein offers street chic.

24. If Saks promotes gift cards, then either Macy’s or Bloomingdale’s puts on a fashion show.

25. Either Rado does not make a sapphire watch or if Movado makes one then so does Pulsar.

26. If either Renée Zellweger or Michelle Pfeiffer accepts a dramatic role, then neither Charlie Sheen nor Ethan Hawke will make an action film.

27. If Kate Winslet and Jessica Biel do a comedy, then either Forest Whitaker will make a documentary or Paris Hilton will do a skin flick.
2.2 Truth Functions Notes

We can express the truth functionality of the five connectives by showing in a list or table exactly how the connectives render the truth value of a molecular proposition computable from the truth values of its components. In order to do so, we need to introduce the idea of a statement variable: a variable that can represent any statement or proposition. Statement variables are represented by lower case letters, such as “p” and “q.” When statement variables are combined by means of connectives, we have statement forms. For example, “p,” “p v q,” and “p • q” are statement forms. When we substitute any propositions uniformly for the statement variables in a statement form, a statement is produced. Such a statement is called a substitution instance of the statement form.

With the notions statement variable and statement form, we can express the truth functionality of the connectives. For instance, consider negation. We may express its truth functionality thusly.

<table>
<thead>
<tr>
<th>p</th>
<th>~p</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

This table means that, regardless of what proposition is used to replace “p,” if it is true, then its negation is false (regardless of its meaning), and if it is false, then its negation is true (again regardless of its meaning).

<table>
<thead>
<tr>
<th>p q</th>
<th>p v q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T T</td>
<td>T</td>
</tr>
<tr>
<td>T F</td>
<td>T</td>
</tr>
<tr>
<td>F T</td>
<td>T</td>
</tr>
<tr>
<td>F F</td>
<td>F</td>
</tr>
</tbody>
</table>

The meaning of this table is that a conjunction of any two propositions is true when and only when both the left-hand conjunct and the right-hand conjunct are true (regardless of the meanings of these conjuncts).

The table of other connectives are:

<table>
<thead>
<tr>
<th>p q</th>
<th>p v q</th>
<th>p q</th>
<th>p ⊃ q</th>
<th>p q</th>
<th>p ≡ q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T T</td>
<td>T</td>
<td>T T</td>
<td>T</td>
<td>T T</td>
<td>T</td>
</tr>
<tr>
<td>T F</td>
<td>T</td>
<td>T F</td>
<td>F</td>
<td>T F</td>
<td>F</td>
</tr>
<tr>
<td>F T</td>
<td>T</td>
<td>F T</td>
<td>F</td>
<td>F T</td>
<td>F</td>
</tr>
<tr>
<td>F F</td>
<td>F</td>
<td>F F</td>
<td>T</td>
<td>F F</td>
<td>T</td>
</tr>
</tbody>
</table>

The calculation may be expressed as follows:

\[(A \lor B) \supset C\]

In performing such calculations, the important thing to remember is to work from inside out, from simpler to more complex.
2.2 Truth Functions Exercises

I. Identify the main operator in the following propositions:

1. \( \sim (A \lor M) \cdot \sim (C \supset E) \)
2. \( (G \cdot \sim P) \supset \sim (H \lor \sim W) \)
3. \( \sim [P \cdot (S \equiv K)] \)
4. \( \sim (K \cdot \sim O) \equiv \sim (R \lor \sim B) \)
5. \( (M \cdot B) \lor \sim [E \equiv \sim (C \lor I)] \)
6. \( \sim [(P \cdot \sim R) \supset (\sim E \lor F)] \)
7. \( \sim [S \lor L) \cdot M \supset (C \lor N) \)
8. \( [\sim F \lor (N \lor U)] \equiv \sim H \)
9. \( (F \lor A) \equiv (\sim G \lor H)] \)
10. \( \sim (X \lor T) \cdot (N \lor F) \lor (K \supset L) \)

II. Determine the truth-values of the following symbolized statements. Let A, B, and C be true and X, Y, and Z be false. Circle your answer.

1. \( A \cdot X \)
2. \( B \cdot \sim Y \)
3. \( X \lor \sim Y \)
4. \( \sim C \lor Z \)
5. \( B \supset \sim Z \)
6. \( Y \supset \sim A \)
7. \( \sim X \supset Z \)
8. \( B \equiv Y \)
9. \( \sim C \equiv Z \)
10. \( \sim (A \cdot \sim Z) \)
11. \( \sim B \lor (Y \supset A) \)
12. \( A \supset \sim (Z \lor \sim Y) \)
13. \( (A \cdot Y) \lor (\sim Z \cdot C) \)
14. \( \sim (X \lor \sim B) \cdot (\sim Y \lor A) \)
15. \( (Y \supset C) \cdot \sim (B \supset \sim X) \)
16. \( (C \equiv \sim A) \lor (Y \equiv Z) \)
17. \( \sim (A \cdot \sim C) \supset (\sim X \supset B) \)
18. \( \sim [(B \lor \sim C) \cdot \sim (X \lor \sim Z)] \)
19. \( \sim [(X \lor C) \equiv \sim (B \supset Z)] \)
20. \( (X \supset Z) \lor [(B \equiv \sim X) \cdot \sim (C \lor \sim A)] \)
21. \( [(\sim X \lor Z) \supset (\sim C \lor B)] \cdot [(\sim X \cdot A) \supset (\sim Y \cdot Z)] \)
22. \( \sim [(A \equiv X) \lor (Z \equiv Y)] \lor [(\sim Y \supset B) \cdot (Z \supset C)] \)
23. \( [(B \cdot \sim C) \lor (X \cdot \sim Y)] \supset \sim [(Y \cdot \sim X) \lor (A \cdot \sim Z)] \)
24. \( \sim \{[(C \lor \sim B) \cdot (Z \lor \sim A)] \cdot \sim [(B \lor Y) \cdot (\sim X \lor Z)]\} \)
25. \( (Z \supset C) \supset \{[(\sim X \supset B) \supset (C \supset Y)] \equiv [(Z \supset X) \supset (\sim Y \supset Z)]\} \)
III. When possible, determine the truth-values of the following symbolized statements. Let A and B be true, Y and Z false. P and Q have unknown truth-value. If the truth-value of the statement cannot be determined, write “undetermined.”

1. \( A \lor P \)
2. \( Q \lor Z \)
3. \( Q \land Y \)
4. \( P \land A \)
5. \( P \supset B \)
6. \( Z \supset Q \)
7. \( A \supset P \)
8. \( P \equiv \neg P \)
9. \( (P \supset A) \supset Z \)
10. \( (P \supset A) \equiv (Q \supset B) \)
11. \( (Q \supset B) \supset (A \supset Y) \)
12. \( \neg(P \supset Y) \lor (Z \supset Q) \)
13. \( \neg(Q \land Y) \equiv \neg(Q \lor A) \)
14. \( [(Z \supset P) \supset P] \supset P \)
15. \( [Q \supset (A \lor P)] \equiv [(Q \supset B) \supset Y] \)
2.3 Truth Tables for Propositions Notes

What are truth tables and why do we use them in logic?

Truth tables "give the truth value of a compound proposition for every possible truth value of its simple components. Each line in the truth table represents one such possible arrangement of truth values."

We use truth tables to analyze compound propositions and arguments. For compound propositions we classify arguments based on their truth table results as either tautologous, self-contradictory or contingent. When we compare statements (analyze full arguments) we classify them as either logically equivalent, contradictory, consistent or inconsistent.

What is the process for setting up a truth table?

To calculate the number of lines in a truth table, use this simple formula: \( L = 2^n \). \( L \) is the number of lines and the small "n" is a variable representing the number of simple propositions contained in the statement. Simple propositions are expressions represented by a single letter.

Symbolize the statement you are evaluating.

Determine the number of lines necessary using the formula presented above.

Divide the total number of lines in half and assign the truth-value "T" to the lines beneath the first simple proposition and assign the value "F" to the remaining lines.

For each successive simple proposition, divide the number of "T" values in half again and assign the value "T" to the first half and "F" to the second half. This process sounds far more complicated than it actually is. To see the process in action follow through the sample arguments that follow.

After truth-values have been assigned to each simple proposition, evaluate the proposition working from inside the parenthesis to the outer layers.

When you evaluate the column under the main operator, you will be asked to classify your argument in accordance with the categories below.

You will not be required to do more than a 16 line truth table, so just remember that:

<table>
<thead>
<tr>
<th>Letters</th>
<th>Lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
</tbody>
</table>
2.4 Truth Tables for Arguments Notes

First, we must represent all the propositions of an argument in a single table. That is, a column must be made for each proposition in the argument. If the argument is in English, it must be translated first, letting capital letters stand for each atomic proposition in the argument.

For example, the argument “If John is happy, then Same is happy; John is happy; therefore, Same is happy” may be represented by:

\[ J \supset S \]
\[ J \]
\[ \sim \]
\[ S \]

Its truth table may be represented by:

<table>
<thead>
<tr>
<th>J</th>
<th>S</th>
<th>J \supset S</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

A truth table may be used to test an argument for validity in a following way. If there is a row in the table in which every premise is true and the conclusion is false, the argument is invalid. If there is no such row, the argument is valid.

For example, the truth table above shows that the argument is valid – there is no row in which all the premises are true and the conclusion is false.
2.4 Truth Tables for Arguments Exercises

Translate the following arguments into symbolic form. Then determine whether each is valid or invalid by constructing a truth table for each.

1. If national elections deteriorate into TV popularity contests, then smooth talking morons will get elected. Therefore, if national elections do not deteriorate into TV popularity contests, then smooth-talking morons will not get elected.

2. Brazil has a huge foreign debt. Therefore, either Brazil or Argentina has a huge foreign debt.

3. If fossil fuel combustion continues at its present rate, then a greenhouse effect will occur. If a greenhouse effect occurs, then world temperatures will rise. Therefore, if fossil fuel combustion continues at its present rate, then world temperatures will rise.

4. If there are dried-up riverbeds on Mars, then water once flowed on the Martian surface. There are dried-up riverbeds on Mars. Therefore, water once flowed on the Martian surface.

5. If high school graduates are deficient in reading, they will not be able to compete in the modern world. If high school graduates are deficient in writing, they will not be able to compete in the modern world. Therefore, if high school graduates are deficient in reading, then they are deficient in writing.

6. The disparity between rich and poor is increasing. Therefore, political control over economic equality will be achieved only if restructuring the economic system along socialist lines implies that political control over economic equality will be achieved.

7. Einstein won the Nobel Prize either for explaining the photoelectric effect or for the special theory of relativity. But he did win the Nobel Prize for explaining the photoelectric effect. Therefore, Einstein did not win the Nobel Prize for the special theory of relativity.

8. If microchips are made from diamond wafers, then computers will generate less heat. Computers will not generate less heat and microchips will be made from diamond wafers. Therefore, synthetic diamonds will be used for jewelry.

9. Either the USS Arizona or the USS Missouri was not sunk in the attack on Pearl Harbor. Therefore, it is not the case that either the USS Arizona or the USS Missouri was sunk in the attack on Pearl Harbor.

10. If racial quotas are adopted for promoting employees, then qualified employees will be passed over; but if racial quotas are not adopted, then prior discrimination will go unaddressed. Either racial quotas will or will not be adopted for promoting employees. Therefore, either qualified employees will be passed over or prior discrimination will go unaddressed.
II. Determine whether the following symbolized arguments are valid or invalid by constructing a truth table for each.

1. \( K \supset \sim K \)
   \hline
   \( \sim K \)

2. \( R \supset R \)
   \hline
   \( R \)

3. \( P \equiv \sim N \)
   \hline
   \( N \lor P \)

4. \( \sim (G \cdot M) \)
   \hline
   \( M \lor \sim G \)
   \hline
   \( \sim G \)

5. \( K \equiv \sim L \)
   \hline
   \( \sim (L \cdot \sim K) \)
   \hline
   \( K \supset L \)

6. \( Z \)
   \hline
   \( E \supset (Z \supset E) \)

7. \( \sim (W \cdot \sim X) \)
   \hline
   \( \sim (X \cdot \sim W) \)
   \hline
   \( X \lor W \)

8. \( C \equiv D \)
   \hline
   \( E \lor \sim D \)
   \hline
   \( E \supset C \)

9. \( A \equiv (B \lor C) \)
   \hline
   \( \sim C \lor B \)
   \hline
   \( A \supset B \)

10. \( J \supset (K \supset L) \)
    \hline
    \( K \supset (J \supset L) \)
    \hline
    \( (J \lor K) \supset L \)

11. \( \sim (K \equiv S) \)
    \hline
    \( S \supset \sim (R \lor K) \)
    \hline
    \( R \lor \sim S \)

12. \( E \supset (F \cdot G) \)
    \hline
    \( F \supset (G \supset H) \)
    \hline
    \( E \supset H \)

13. \( A \supset (N \lor Q) \)
    \hline
    \( \sim (N \lor \sim A) \)
    \hline
    \( A \supset Q \)

14. \( G \supset H \)
    \hline
    \( R \equiv G \)
    \hline
    \( \sim H \lor G \)
    \hline
    \( R \equiv H \)

15. \( L \supset M \)
    \hline
    \( M \supset N \)
    \hline
    \( N \supset L \)
    \hline
    \( L \lor N \)
2.5 Indirect Truth Tables Notes

Indirect truth tables provide a shortcut method for testing argument validity. In order to set up this shortcut method, we have to consider all possible situations in which all premises could be true and the conclusion false.

Testing arguments for validity:

The process for testing arguments for validity is described in the section. The method is:

Look at the conclusion and determine the ways in which the main operator could evaluate "false."

Assign truth values of false to the conclusion and true for the premises for each situation in which the main operator in the conclusion could evaluate false.

Working backwards from the truth values generated by evaluating the conclusion false, deduce the truth values for the remaining simple propositions and premises.

If you do not encounter a contradiction while assigning truth values to the premises, then it is possible for all premises to be true and the conclusion false. Thus, the argument is invalid.

If a contradiction is produced in the attempts to assign truth values to the premises, circle the contradiction and declare the argument valid.

For arguments with multiple ways of evaluating false:

You must generate a contradiction on every line for the argument to be valid.

You can stop evaluating on a single line when you produce a contradiction.

If you fail to produce a contradiction on any line, (i.e., your deductions produce true premises and a false conclusion), you can stop evaluating the argument. It is invalid.
2.5 Indirect Truth Tables Exercises

Use indirect truth tables to determine whether the following arguments are valid or invalid.

1. B ⊃ C
   ~C
   -------
   ~B

2. ~E ∨ F
   ~E
   -------
   ~F

3. P ⊃ (Q • R)
   R ⊃ S
   -------
   P ⊃ S

4. ~(I ≡ J)
   -------
   ~(I ⊃ J)

5. W ⊃ (X ⊃ Y)
   X ⊃ (Y ⊃ Z)
   -------
   W ⊃ (X ⊃ Z)

6. A ⊃ (B ∨ C)
   C ⊃ (D • E)
   ~B
   -------
   A ⊃ ~E

7. G ⊃ H
   H ⊃ I
   ~J ⊃ G
   ~I
   -------
   J

8. J ⊃ (~L ⊃ ~K)
   K ⊃ (~L ⊃ M)
   (L ∨ M) ⊃ N
   -------
   J ⊃ N

9. P • (Q ∨ R)
   (P • R) ⊃ ~(S ∨ T)
   (~S ∨ ~T) ⊃ ~(P • Q)
   -------
   S ≡ T

10. (M ∨ N) ⊃ O
    O ⊃ (N ∨ P)
    M ⊃ (~Q ⊃ N)
    (Q ⊃ M) ⊃ ~P
   -------
    N ≡ O

11. (A ∨ B) ⊃ (C • D)
    (~A ∨ ~B) ⊃ E
   -------
    (~C ∨ ~D) ⊃ E

12. F ⊃ G
    ~H ∨ I
    (G ∨ I) ⊃ J
    ~J
   -------
    ~(F ∨ H)

13. (A ∨ B) ⊃ (C • D)
    (X ∨ ~Y) ⊃ (~C • ~W)
    (X ∨ Z) ⊃ (A • E)
   -------
    ~X